Part B	Problems 1-10 which only require answers.
Part C	Problems 11-20 which require complete solutions.
Test time	150 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D).
Together they give a total of 61 points consisting of
22 E-, 22 C- and 17 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 31 points of which 12 points on at least C-level
B: 41 points of which 5 points on A-level
A: 49 points of which 9 points on A-level

The number of points you can get for a complete solution is stated after each problem. You can also see what knowledge levels (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.
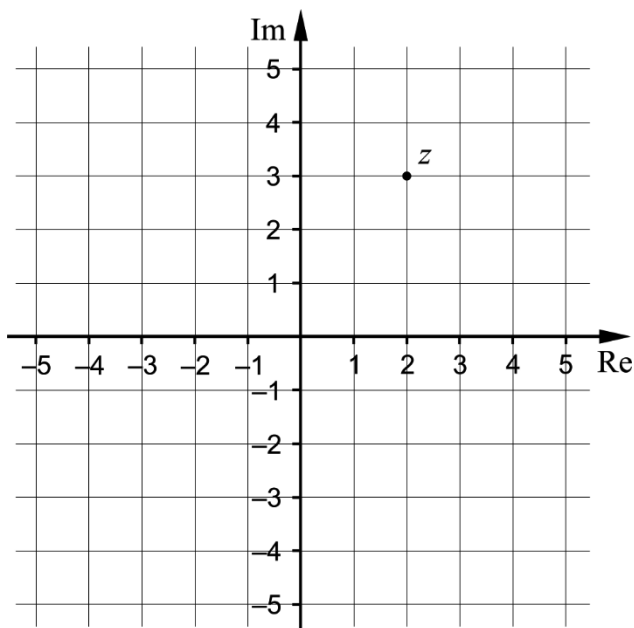
Name: _____

Date of birth: _____

Educational programme: _____

Part B: Digital resources are not allowed. *Only answer is required.* Write your answers in the test booklet.

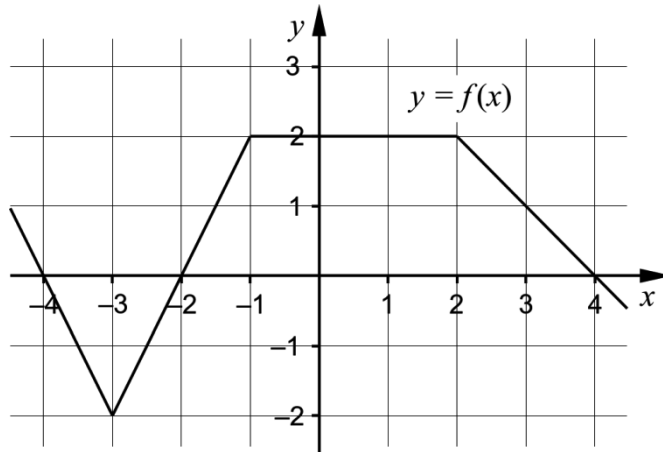1. It holds for the function f that $f(x) = 5\sin 4x + 3$

   a) Find the largest possible value of the function.

   _____ (1/0/0)

   b) Find $f'(x)$.

   _____ (1/0/0)

2. The complex number z is marked in the complex plane.

   a) Mark the number $\overline{z}$ in the complex plane.   (1/0/0)

   b) Find $z \cdot \overline{z}$

   _____ (1/0/0)

3. The figure shows the graph of a function f.

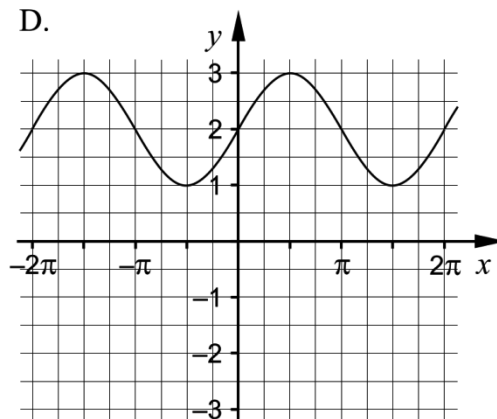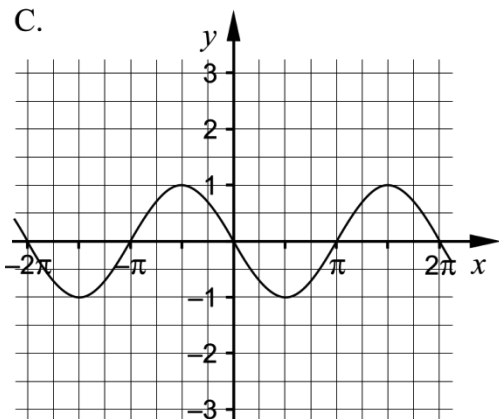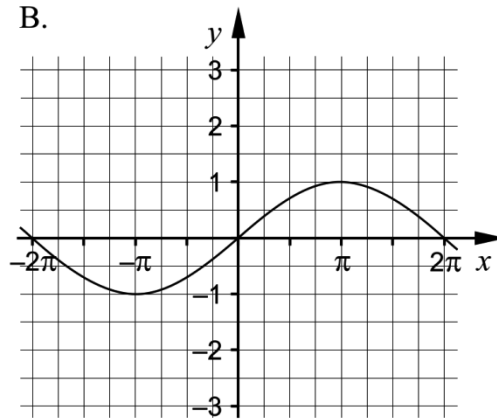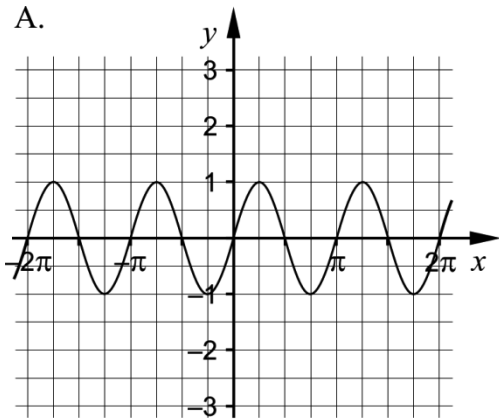Find $\displaystyle\int_{-3}^{0} f(x)\,dx$ _____ (1/0/0)

4. It holds for the complex numbers z and w that

$$z = 7\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) \text{ and } w = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

a) Find $\left|\dfrac{z}{w}\right|$ _____ (1/0/0)

b) Find $\arg\left(\dfrac{z}{w}\right)$ _____ (1/0/0)

5. The figures show the graphs of four trigonometric functions.

A.

B.

C.

D.

a) Match the following three functions with the corresponding graphs A–D.

$y = \sin(x) + 2$ corresponds to graph: _____

$y = \sin(2x)$ corresponds to graph: _____

$y = \sin(x + \pi)$ corresponds to graph: _____ (0/1/0)

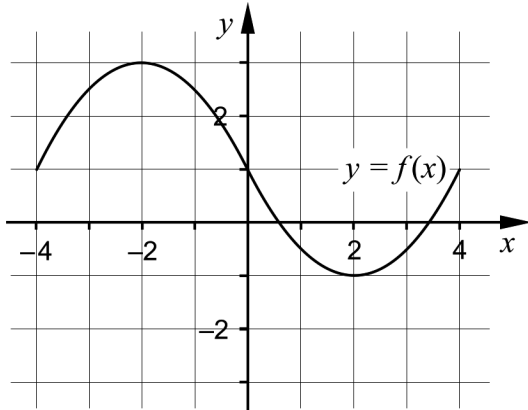b) One of the graphs A–D does not correspond to any of the three functions in a).
Write down a trigonometric function for this graph.

_____ (0/1/0)

6. Determine the constant a so that the polynomial $p(x) = x^5 + 2x^4 - 8x + a$
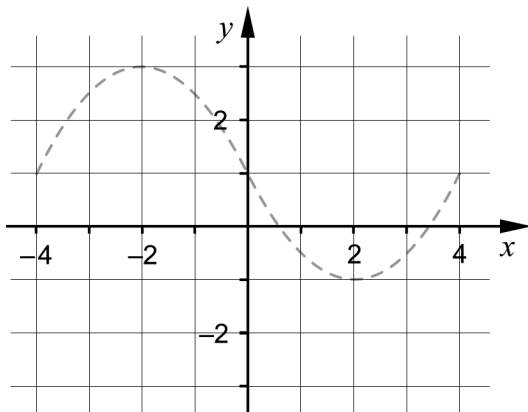is divisible by the factor $(x - 1)$.

_____ (0/1/0)

7. In the coordinate system, the curve $y = f(x)$ is drawn on the interval $-4 \leq x \leq 4$

Use the coordinate system below to sketch the curve $y = |f(x)|$

on the interval $-4 \leq x \leq 4$

To make your sketching easier, the curve $y = f(x)$ has been drawn with a dashed line.

(0/1/0)

8. $z_1 = \cos 35° + i \sin 35°$ is a root of the equation $z^9 = w$.

Find another root of the same equation. _____ (0/0/1)

9. Which of the alternatives A–H is the best approximate value of

$$\frac{\sin(\frac{\pi}{3} + 0.01) - \sin\frac{\pi}{3}}{0.01} \ ?$$

A. 0
B. 0.01
C. 0.5
D. 1
E. 2
F. 10
G. 50
H. 100   _____ (0/0/1)

10. Give an example of a function f with the derivative

$$f'(x) = 24x(x^2 + 1)^5$$

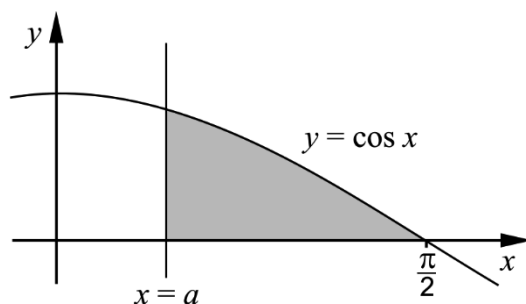   _____ (0/0/1)

Part C: Digital resources are not allowed. Write your solutions on separate sheets of paper.

11. Calculate $\dfrac{3+5i}{1+i}$. Give your answer in the form $a+bi$. (2/0/0)

12. Solve the equation $\sin 3x = \dfrac{\sqrt{3}}{2}$ (2/1/0)

13. Show that $\dfrac{1-\cos^2 x}{\sin x \cos x} = \tan x$ for all x where the expressions are defined. (2/0/0)

14. The shaded region in the figure is bounded by the curve $y = \cos x$, the x-axis and the line $x = a$, where $0 < a < \dfrac{\pi}{2}$

Determine a so that the area of the region is $\dfrac{1}{2}$ a.u. (2/1/0)

15. The revenue when selling a product is given by
$$I(p) = 2000\, p \cdot e^{-0.05p}$$
where I is the revenue in SEK/day and p is the price of the product in SEK.

Decide whether there is a price p which gives a maximum revenue, and if so, what is this price? (0/3/0)

16. Parham works with the differential equation $y'' + 8y = 6y'$. He concludes that $y = 4e^{2x}$ is a solution to the equation and shows the result to Aida. Aida studies the equation and says that it cannot be true. She claims that the numbers 4 and 2 have accidentally changed places in Parham's solution, because the solution should be $y = 2e^{4x}$ according to Aida.

Investigate whether either of them is wrong. (0/2/0)

17. The curve $y = h - x^2$, where h is a positive constant, bounds together with the coordinate axes a region in the first quadrant.

Find h so that the area of the region is $\dfrac{16}{3}$ a.u. (0/1/1)

18. Show that $\sin 345° = \dfrac{\sqrt{2} - \sqrt{6}}{4}$ (0/0/2)

19. Determine the smallest value that the function $y = e^{\sin x \cos x}$ can have. Give an exact answer. (0/0/2)

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20. The functions f_1, f_2, f_3 and f_4 are defined as follows:

$f_1(x) = \dfrac{1}{x} + x$	$f_2(x) = \dfrac{1}{x} + 3x$
$f_3(x) = \dfrac{1}{3x} + x$	$f_4(x) = \dfrac{1}{3x} + 3x$

The figures below show the graphs A−D of the functions for $x > 0$
All graphs are drawn to the same scale in the coordinate systems.

A.

B.

C.

D.
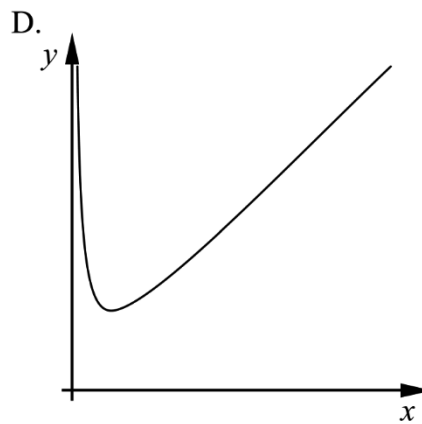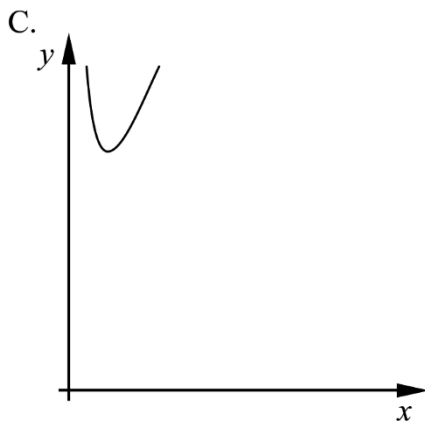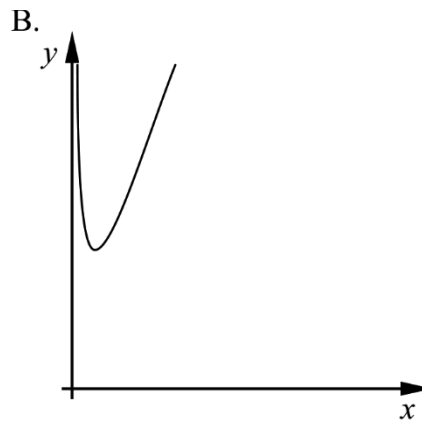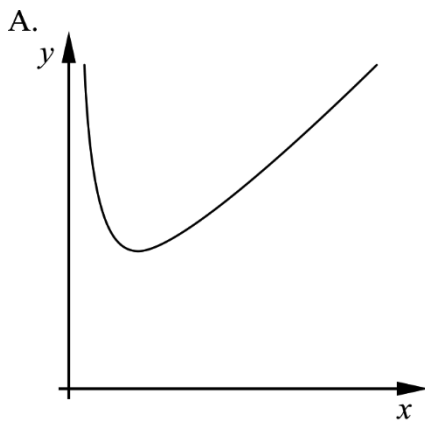
Match each function $f_1 - f_4$ with the corresponding graph A−D. Justify
your answer. (0/0/2)