| | |
|---|---|
| **Part B** | Problems 1-9 which only require answers. |
| **Part C** | Problems 10-14 which require complete solutions. |
| **Test time** | 120 minutes for Part B and Part C together. |
| **Resources** | Formula sheet and ruler. |

**Level requirements**

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 53 points consisting of 22 E-, 18 C- and 13 A-points.

Level requirements for test grades
E: 14 points
D: 22 points of which 6 points on at least C-level
C: 29 points of which 10 points on at least C-level
B: 37 points of which 4 points on A-level
A: 43 points of which 7 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled *"Only answer required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.
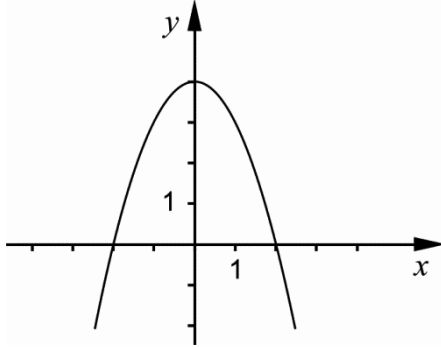
**Write your name, date of birth and educational programme on all the sheets you hand in.**

Name: _____

Date of birth: _____

Educational programme: _____

**Part B:** Digital resources are not allowed. *Only answer is required.* Write your answers in the test booklet.

1.  Calculate $f(3)$ if $f(x) = 9 + x^2$       _____ (1/0/0)

2.  The figure shows the graph of the function $y = -x^2 + c$

    

    a)  Use the figure to determine the zeroes of the function.

                                            _____ (1/0/0)

    b)  Use the figure to determine the value of the constant $c$.

                                            _____ (1/0/0)

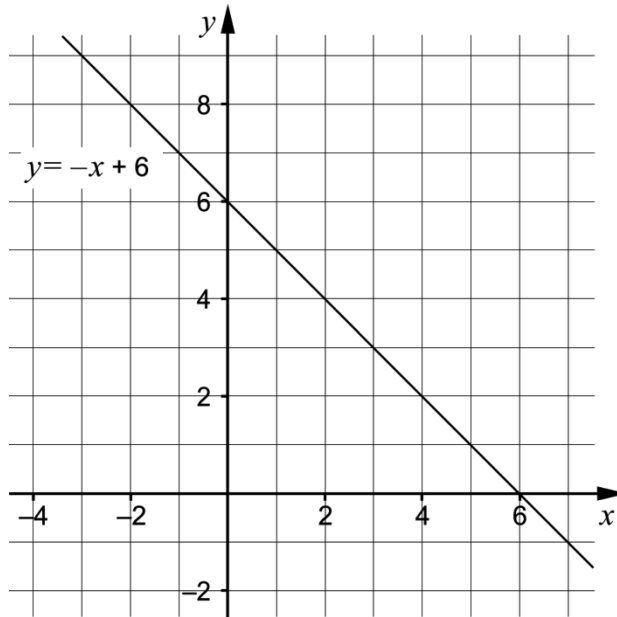3.  Simplify $(x + 5)^2 - 10x$ as far as possible.       _____ (1/0/0)

4.  Solve the equations

    a)  $x^2 - 64 = 0$                             _____ (1/0/0)

    b)  $x^{\frac{1}{2}} = 2$                                _____ (1/0/0)

5.  Calculate $5^{\frac{1}{3}} \cdot 5^{\frac{5}{3}}$                 _____ (1/0/0)

**6.** A system of linear equations consists of two equations. The graph of one of the equations is drawn in the coordinate system.



a) The graph of the other equation has a gradient $k = 0.5$
   Draw the graph of this equation so that the system of linear equations
   has the solution $\begin{cases} x = 2 \\ y = 4 \end{cases}$ (1/1/0)

b) Specify the system of linear equations which is now drawn in the coordinate system.

   _____ (0/1/0)

**7.** Below are three equations and four statements.

$(x + 2)^2 = (x - 2)^2$

$(x + 2)(x - 2) = (2 + x)(2 - x)$

$(x + 2)^2 = (x + 2)^2$

The equation does not have a solution

The equation has one solution

The equation has two solutions

The equation has an infinite number of solutions

Draw a line from each one of the equations to the correct statement. (0/1/1)

**8.**   The figure shows the graph of the function  $f$
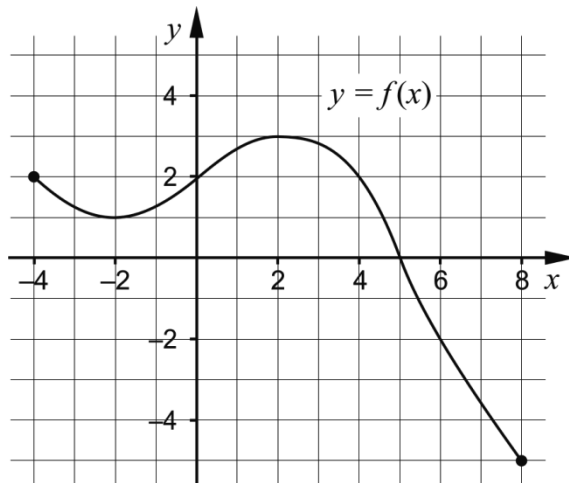


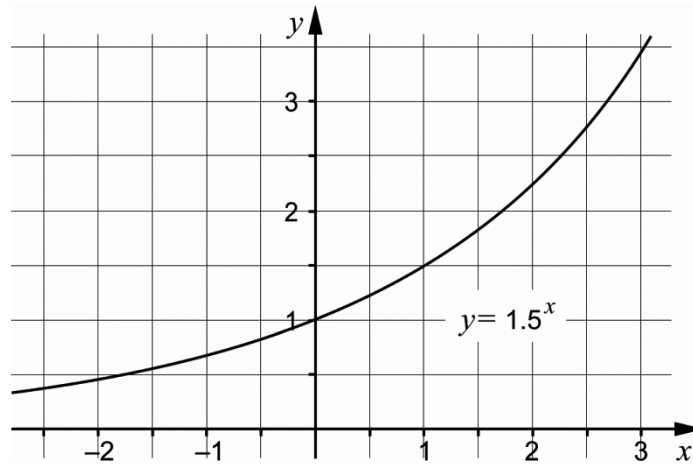a)   Which of the alternatives A-F represents the range of the function?

A.   $-5 \le y \le 2$

B.   $-5 \le x \le 2$

C.   $-4 \le y \le 8$

D.   $-4 \le x \le 8$

E.   $-5 \le y \le 3$

F.   $-5 \le x \le 3$

_____ (0/1/0)

b)   Determine  $f(a)$  when  $f(a+1) = -2$       _____ (0/0/1)

**9.** The figure shows the graph of the exponential function $y = 1.5^x$



$y = 1.5^x$

Use the graph to solve the following equations.

a)  $1.5^x = 3$  _____ (1/0/0)

b)  $1.5^x \cdot 1.5^{-2x} = 3$  _____ (0/0/1)

> **Part C**: Digital resources are not allowed. Do your solutions on separate sheets of paper.

**10.** Solve the equation $x^2 - 12x + 20 = 0$ algebraically. (2/0/0)

**11.** Sonny is visiting Umeå. During his visit he plans to travel on the local bus. On the bus company's web page he can read about ticket prices for youths aged 7-19.

| Ticket price youths aged 7-19 | | |
|---|---|---|
| Single trip | | SEK 13/trip |
| Travel card | Price for a card without prepaid trips | SEK 25 |
| | Price for each prepaid trip | SEK 9/trip |

When buing a card that are prepaid with $x$ number of trips the total cost will be SEK $y$.

a) Specify a linear relation between the total cost SEK $y$ and the number of trips $x$. *Only answer required* (1/0/0)

Sonny is thinking of buying a travel card.

b) How many trips does Sonny at least have to make if it is going to be worth buying a travel card instead of buying single tickets? (2/0/0)

**12.** Bengt in Boda is going to build a rectangular pasture for his horses on the fields bordering to Lake Viggaren. He has 180 m of fencing which he will use when building the pasture. No fencing is needed along the lake, see the figure below.
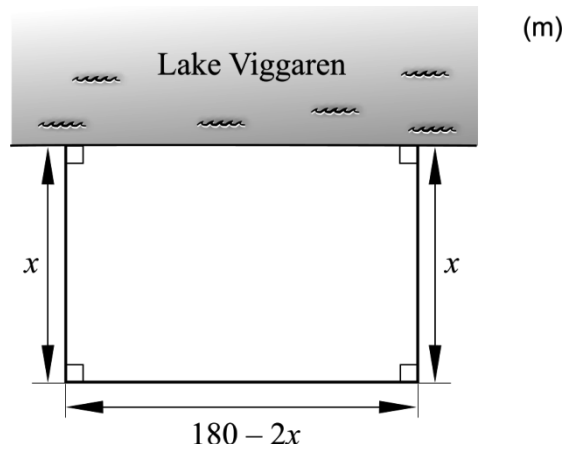


Write down an expression for the area of the pasture and decide what dimensions the pasture should have in order for the area to be as large as possible. (1/3/0)

**13.** What are the possible values of the constant $m$ if the graphs of the functions
$y = x^2 + 3.7$ and $y = 2x + m$
should not intersect? (0/0/2)

**14.** The corners of a right-angled triangle has the coordinates
$(-2, 0)$, $(6, 0)$ och $(0, a)$ where $a > 0$
Find the exact value of $a$. (0/0/3)

| **Part D** | Problems 15-23 which require complete solutions. |
| **Test time** | 120 minutes. |
| **Resources** | Digital resources, formula sheet and ruler. |

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**Write your name, date of birth and educational programme on all the sheets you hand in.**

Name: _____

Date of birth: _____

Educational programme: _____

Part D: Digital resources are allowed. Do your solutions on separate sheets of paper.

15. Find the equation of the straight line that passes through the points (4, 3) and (6, 7) (2/0/0)

16. Anna and Stina buy sweets. Anna buys 4 lollies and 12 toffees and pays SEK 32. Stina buys 2 lollies and 4 toffees and pays SEK 13.

"How much is a lolly and a toffee, respectively?" wonders Anna.
"We can find out by solving simultaneous equations," says Stina.

Stina writes down the following simultaneous equations.

$$\begin{cases} 4x + 12y = 32 \\ 2x + 4y = 13 \end{cases}$$

a) What do $x$ and $y$ mean in this context? (1/0/0)

b) Solve the simultaneous equations and determine the price of a lolly and a toffee, respectively. (2/0/0)
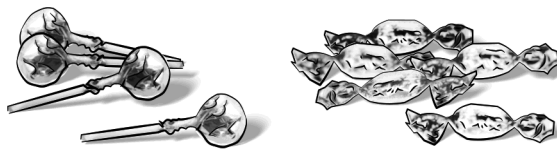
17. A straight line with gradient $k = 3.5$ passes through the point (2, 5) Does the line also pass through a point with the $y$-coordinate $-500$? Justify your answer. (0/1/0)

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**18.** Hjördis is a plumber and runs her own business. She's got more work than she can manage and needs to employ a new person. In her budget for next year she plans to set aside SEK 350 000 to cover both salary and payroll tax for the new employee.

The size of the payroll tax depends on the age of the employee and the monthly salary. See table.

| Age | Payroll tax |
|---|---|
| Age 26 and younger | 15.49% of the salary |
| Age 27 – 65 | 31.42% of the salary |
| Age 66 and older | 10.21% of the salary |

After interviews, Hjördis has decided to employ Anton or Niklas.

Anton who is 24 years old has asked for a monthly salary of SEK 25 000.
Niklas who is 28 years old has asked for a monthly salary of SEK 24 000.
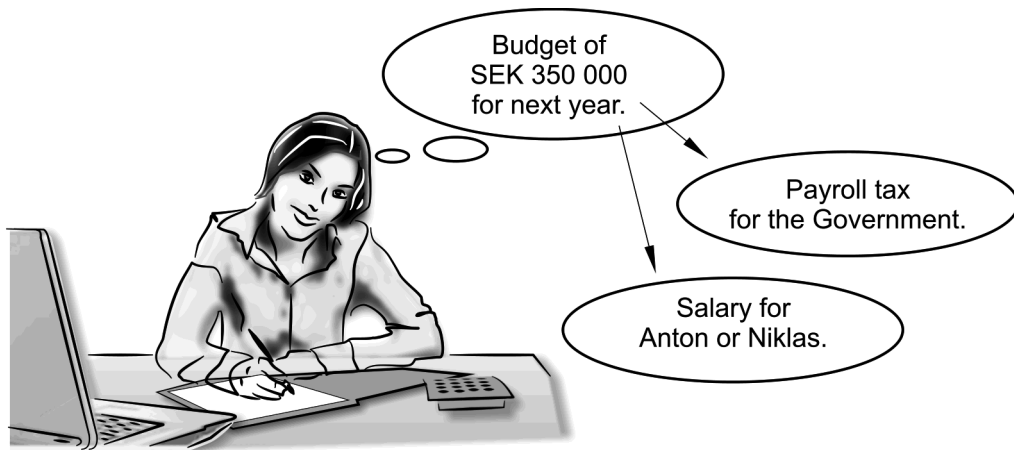


a) Calculate the total cost that Hjördis will have to pay for salaries and payroll taxes for Anton and Niklas respectively. Can Hjördis employ either of them and still be within her budget of SEK 350 000 for next year? (2/0/0)

b) Hjördis's business has a turnover of SEK 2 000 000 per year. With a new employee in the company, her goal is to double the turnover in three years. By what per centage must the turnover increase every year on average? (0/2/0)

**19.** Determine the constants $a$ and $b$ so that the simultaneous equations $\begin{cases} y = ax + 1 \\ a = y - 3x \end{cases}$

have solutions $x = 3$ and $y = 2b$ (0/2/0)

**20.** Adelina and Linda are practising rounders. Adelina hits the ball with a bat and Linda practises catching fly balls, that is catching the ball before it hits the ground.

On one occasion, the trajectory of the ball can be described by the function
$y = -0.10x^2 + 2x + 1$
$y$ is the ball's height above the ground.
$x$ is the distance, in metres, along the ground from where the ball was hit.
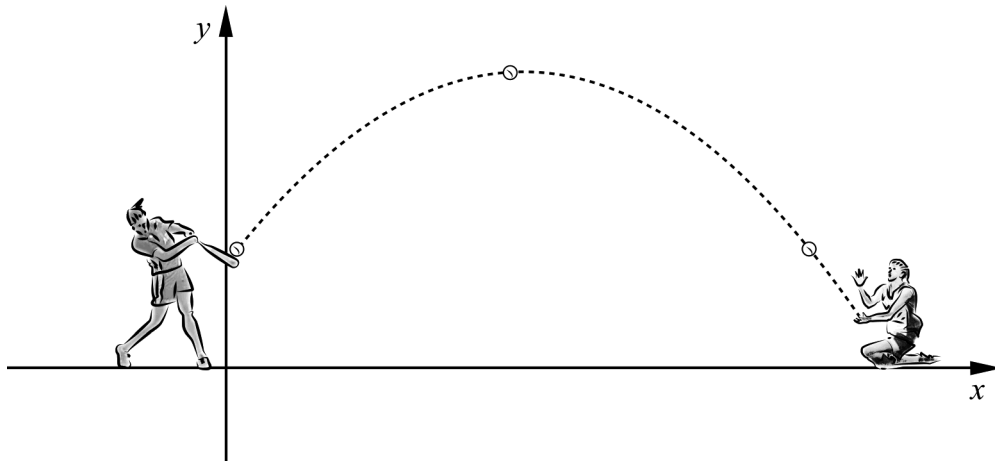


How far from where the ball was hit is Linda if she catches the ball 0.80 m above the ground? (0/3/0)

**21.** It holds for the function $f$ that $f(x) = x^2$

Find all values of $a$ so that $f(2a) = a$ (0/2/0)

**22.** It holds for the numbers $x$ and $y$ that $x^2 + 2xy + y^2 = 9$

Show algebraically that all solutions to the relation can be described by two straight lines. (0/1/1)

23. The company "Lexelius Jump and Bounce" sells rectangular trampolines. On every trampoline, the long side is twice as long as the short side. The company recommends that there is a 2.0 m wide safety zone around the trampoline and that the area of the safety zone should be at least three times as large as the area of the trampoline.



Safety zone      2 m

2 m
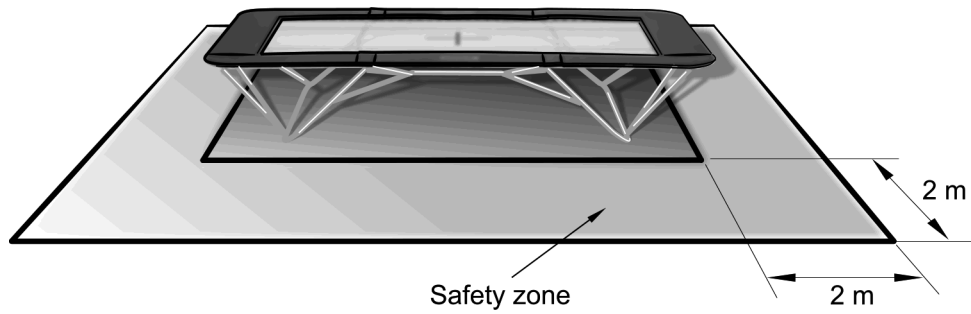
Calculate the dimensions of a trampoline that has a 2.0 m wide safety zone and where the area of the safety zone is three times as large as the area of the trampoline. (0/0/4)