

Part D	Problems 20-27 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 62 points consisting of 22 E-, 23 C- and 17 A-points.

Level requirements for test grades

E: 15 points

D: 24 points of which 7 points on at least C-level

C: 32 points of which 13 points on at least C-level

B: 41 points of which 5 points on A-level

A: 49 points of which 9 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled “*Only answer required*” you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures and show how you use your digital resources.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

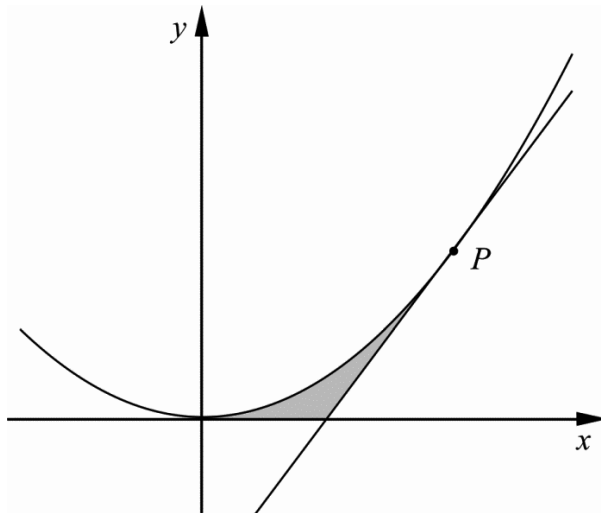
Part D: Digital resources are allowed. Write your solutions on separate sheets of paper.

20. The temperature in Haparanda was measured over 24 hours in July. According to a simplified model, the temperature during this time can be described by the relation $y = 15 + 5\sin(0.26x)$ where y °C is the temperature and x is the number of hours after 08.00.

Calculate the rate of change for the temperature at the time 12.00. (2/0/0)

21. Determine the number a so that $y = a \cdot e^{2x}$ is a solution to the differential equation $y' + y = e^{2x}$ (2/0/0)

22. The figure below shows the parabola $f(x) = x^2$ and the line $g(x) = 4x - 4$. The line touches the parabola at the point P . The parabola, the line and the x -axis together enclose the shaded region in the figure.



Calculate the area of the shaded region. (2/1/0)

23. Write down a function that has two vertical asymptotes. *Only answer is required* (0/2/0)

24. As a part of a quality control of a bakery a number of cinnamon buns are weighed. The quality control shows that the weight is normally distributed with an average weight of 120 grams and a standard deviation of 4.0 grams.

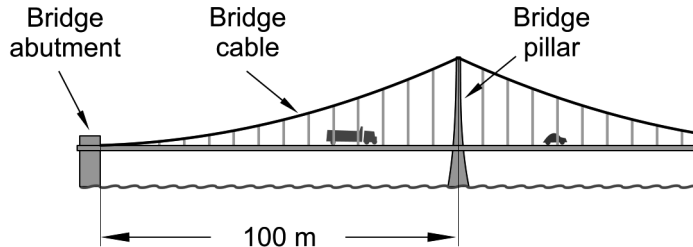
How many cinnamon buns can be expected to have a weight between 115 grams and 130 grams out of 450 cinnamon buns baked in one day? (0/2/0)

25. The shape of the bridge cable in the figure below can according to a simplified model be described by the function

$$f(x) = 0.040x^{3/2} \text{ within the interval } 0 \leq x \leq 100, \text{ where}$$

f is the height above the roadway in metres and

x is the distance in metres from the bridge abutment, measured along the roadway.



Facts:

The length s of a curve $y = f(x)$ within the interval $a \leq x \leq b$ is given by the relation

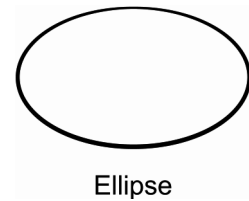
$$s = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

Calculate the length of the bridge cable between the bridge abutment and the bridge pillar. (0/2/0)

26. The equation of a circle with its centre at the origin and radius 1 is
 $x^2 + y^2 = 1$

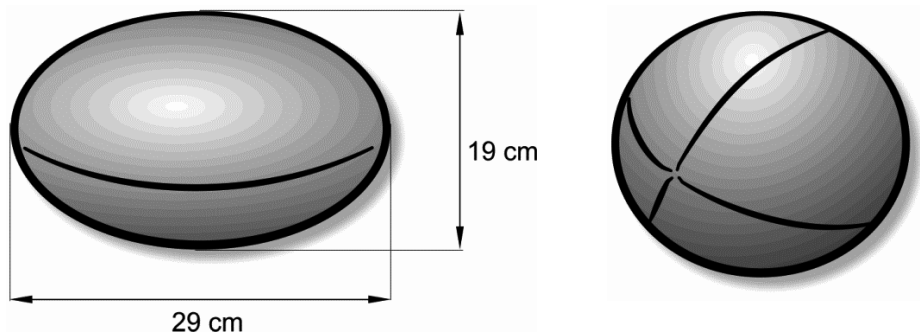
Similarly, the equation of an ellipse with its centre at the origin and that intersects the axis at $(\pm a, 0)$ and $(0, \pm b)$ is

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$



When this type of ellipse rotates around the x -axis one gets an ellipsoid. The ball used in rugby has the shape of an ellipsoid.

One type of ball approved for rugby games has the measures given in the figure below.



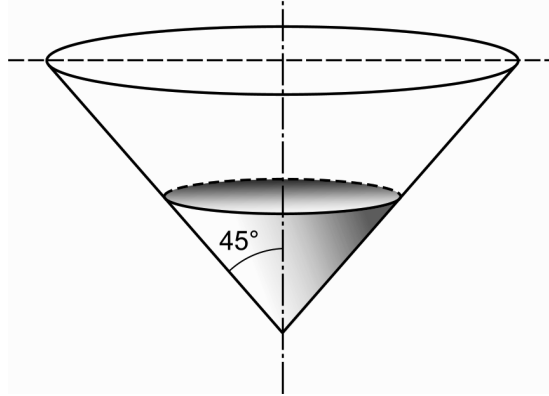
Calculate the volume of this ball.

(0/0/3)

27. Lasse and Marcus are going to solve the following problem:

A container has the shape of a cone as can be seen from the figure. The container is initially empty. Water is added at a velocity of $(25 + 0.2t)$ litres/min, where t is the time in minutes from the start of the filling.

At what speed does the water level of the container raise when it is 7.0 dm?



Lasse first calculates that it takes 13.6 minutes for the water level to reach 7.0 dm.

Marcus will use Lasse's results to solve the rest of the problem. Marcus starts by denoting the water level with h and determines the volume of the container expressed in h . He then calculates the requested velocity.

- a) Start with Lasse's results and perform Marcus's part of the solution. (0/1/1)
- b) Show that Lasse's calculations are correct, that is, that the water level is 7.0 dm after 13.6 minutes. (0/1/2)