Part B	Problems 1-11 which only require answers.
Part C	Problems 12-16 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 65 points consisting of 24 E-, 23 C- and 18 A-points.

Level requirements for test grades
E: 18 points
D: 28 points of which 8 points on at least C-level
C: 36 points of which 15 points on at least C-level
B: 46 points of which 7 points on A-level
A: 55 points of which 12 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: ______________________________

Date of birth: ______________________________

Educational programme: ______________________________

Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

1. Determine $f'(x)$ if

a) $f(x) = 4x^3 + 7x + 2$ $f'(x) =$ ________________ (1/0/0)

b) $f(x) = e^{2x}$ $f'(x) =$ ________________ (1/0/0)

2. Calculate $\left|3 - 3^2\right|$ ______________________ (1/0/0)

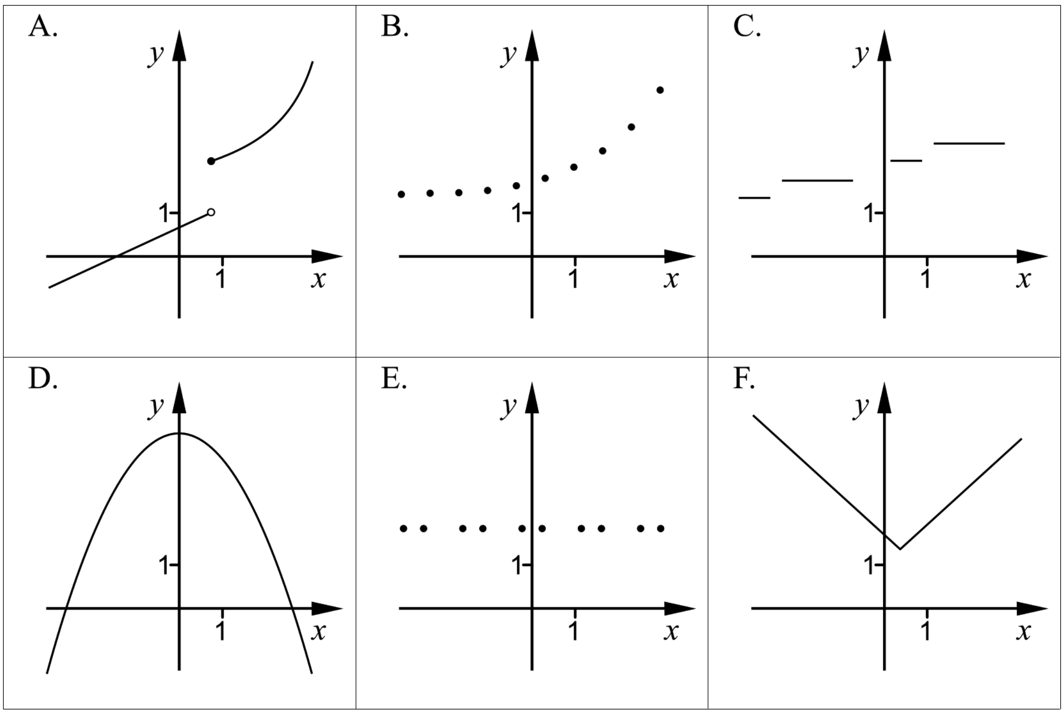
3. The figures show the main characteristics of the graphs of six different functions.

a) Two of the figures A-F show a graph of a discrete function. Which two?

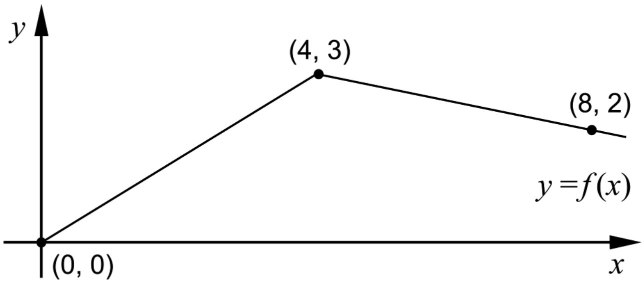
______________________ (1/0/0)

b) Two of the figures A-F show a graph of a function which is continuous for all *x*. Which two?

______________________ (1/0/0)



4. The figure shows the graph of the function f.



a) Determine $\int_0^4 f(x)\,dx$ ______________________ (1/0/0)

b) Determine $f'(5)$ ______________________ (1/0/0)

5. Simplify the expressions as far as possible.

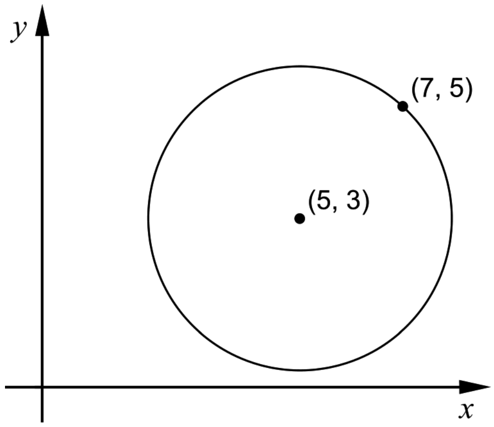
a) $x(7+x)(7-x)+x^3$ ______________________ (1/0/0)

b) $\left(\frac{1}{x}+\frac{1}{x}\right)^{-1}$ ______________________ (0/1/0)

c) $\frac{2}{x-2}+\frac{x}{2-x}$ ______________________ (0/1/0)

6. The equation of a circle can be written $(x-a)^2+(y-b)^2=r^2$.
The point (7, 5) lies on a circle with its centre at (5, 3), see figure.

Determine a, b and r for this circle.



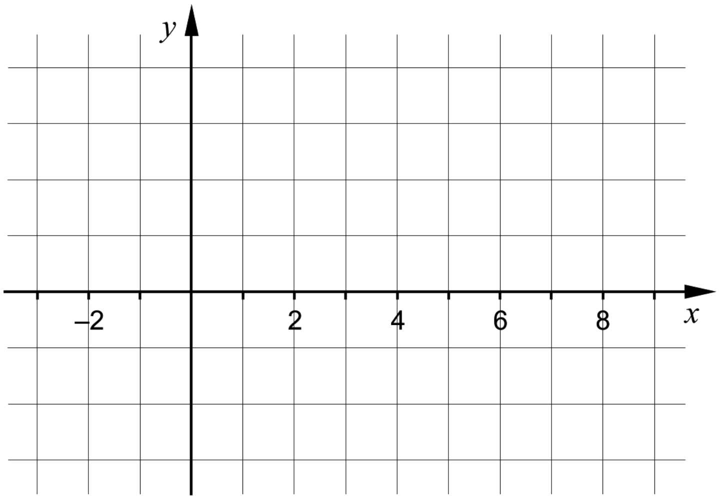
$a =$ ________ $b =$ ________ (1/0/0)

$r =$ ____________________ (0/1/0)

7. It holds for a polynomial function f that the derivative has only two zeroes. The table shows the sign of the derivative for some different values of x.

x	-2	0	2	5	7
$f'(x)$	$-$	0	$+$	0	$+$

Sketch a possible graph of the function f in the coordinate system below. (0/2/0)



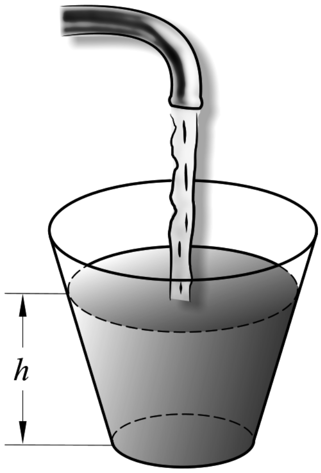
8. There are several rational expressions that satisfy the following conditions:

- The expression has the value 0 only when $x = -5$
- The expression is not defined for $x = 10$

Give an example of a rational expression that satisfies both conditions.

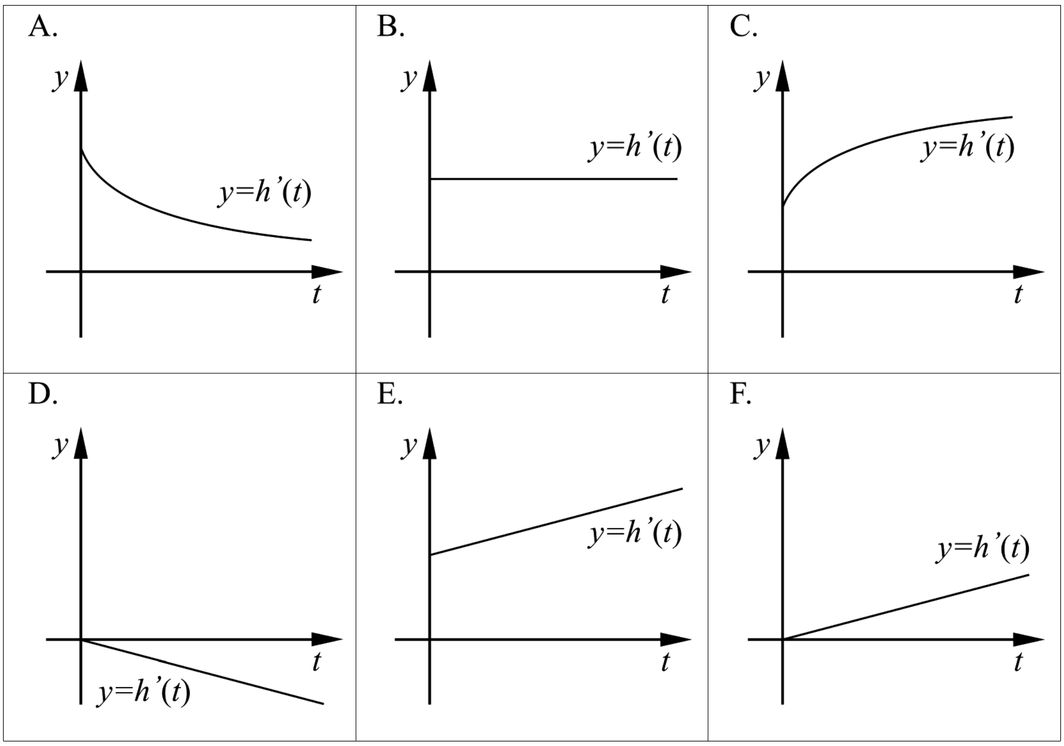
_______________________ (0/2/0)

9. The figure shows how a glass is filled with water. The glass is narrower at the bottom. The water pours out of the tap at a constant speed. The height of the water surface *h* above the bottom of the glass is a function of time *t.*



Which of the graphs A-F *best* describes the derivative $h'(t)$ during the time the glass is filled?

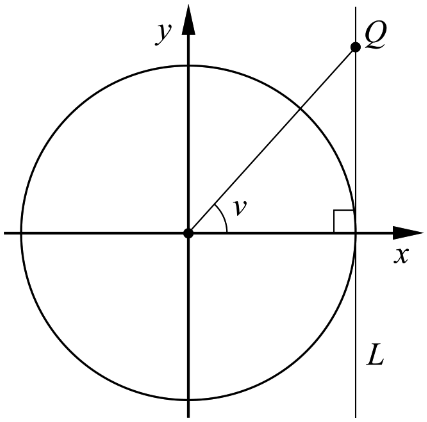
______________________ (0/1/0)



10. Give an example of a function f that is not constant and that has the limit 3 when $x \to \infty$.

$f(x) =$ ________________ (0/0/1)

11. The figure below shows a unit circle touched by a line L which is parallel to the y-axis. There is a point Q on the line L that has y-coordinate t. The line segment between the origin and Q forms the angle v with the x-axis. It holds for the angle v that $0° < v < 90°$.



Determine $\cos v$ expressed in t. ______________________ (0/0/1)

Part C: Digital resources are not allowed. Do your solutions on separate sheets of paper.

12. Olle and Olga sell chanterelles and are considering raising the kilo price of the chanterelles in order to increase the daily income. They have found that the daily income as a function of the increase in price is given by

$f(x) = -0.1x^2 + 5x + 3000$

where $f(x)$ is the daily income in SEK and x is the increase in price in SEK/kg.

Calculate, by using the derivative, what increase in price x that gives the largest daily income. (2/0/0)

13. Calculate

a) $\int_1^2 4x^3 \, dx$ (2/0/0)

b) $\int_2^4 \frac{2}{x^2} \, dx$ (0/2/0)

14. Determine $f''(4)$ if $f(x) = \frac{\sqrt{x}}{2}$.
Give the answer on the simplest form. (0/2/0)

15. What must be true in order for the line $y = f(x)$ to touch the curve $y = g(x)$ at the point where $x = a$? (0/0/2)

16.

A unit fraction is a fraction where the numerator is 1 and the denominator is a positive integer, that is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and so on. The Egyptians used unit fractions in their calculations. Instead of writing $\frac{5}{6}$ they wrote the fraction as a sum of different unit fractions: $\frac{1}{2}+\frac{1}{3}$

The fraction $\frac{2}{3}$ can be written as the sum of three unit fractions that satisfies the conditions:

- The second unit fraction has a denominator which is 3 times as large as the numerator of the first unit fraction.
- The third unit fraction has a numerator which is 1 less than the numerator of the first unit fraction.

Write down an equation and show by solving this that there is only one way to write the fraction $\frac{2}{3}$ as a sum of three unit fractions, if the conditions are satisfied. (0/0/3)