

Part B	Problems 1-11 which only require answers.
Part C	Problems 12-16 which require complete solutions.
Test time	120 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 66 points consisting of 24 E-, 23 C- and 19 A-points.

Level requirements for test grades

E: 17 points

D: 27 points of which 8 points on at least C-level

C: 35 points of which 14 points on at least C-level

B: 45 points of which 6 points on A-level

A: 53 points of which 11 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A-point.

For problems labelled “*Only answer is required*” you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

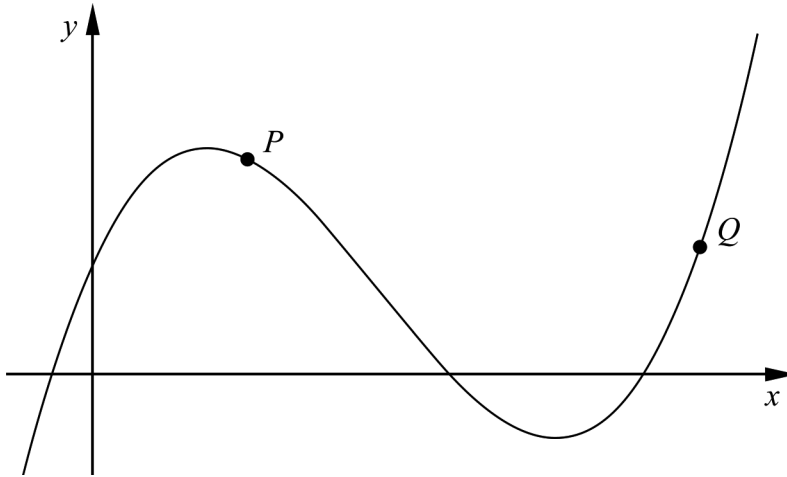
Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____
Date of birth: _____
Educational programme: _____

Part B: Digital resources are not allowed. *Only answer is required.* Write your answers in the test booklet.

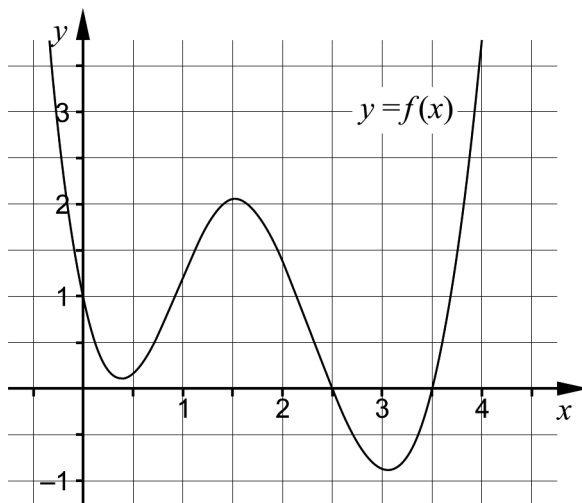
1. It holds for the function f that $f(x) = 3x^4 - 12x$
 Determine $f'(x)$ _____ (1/0/0)

2. The figure shows the graph of a cubic function.



In the figure,

- a) draw a tangent to the curve at point P . (1/0/0)
- b) draw a secant that passes through the point Q . (1/0/0)
3. The figure shows the key features of the graph of a function f .



Solve the equation $f(x) = 0$ _____ (1/0/0)

4. Simplify the expressions as far as possible.

a) $\frac{(x+3)^{10}}{(x+3)^5}$ _____ (1/0/0)

b) $\frac{a}{\frac{1}{2a} + \frac{1}{2a}}$ _____ (0/1/0)

5. The value of a car decreases exponentially according to the relation

$$V(t) = 100\,000 e^{-0.2t}$$

where V is the value in SEK and t is the time in years after the purchase.

Which of the alternatives A-H below represents the rate of change of the value of the car 5 years after the purchase?

- A. $-100\,000 e^{-1}$ SEK
- B. $-100\,000 e^{-1}$ SEK/year
- C. $100\,000 e^{-1}$ SEK
- D. $100\,000 e^{-1}$ SEK/year
- E. $-20\,000 e^{-1}$ SEK
- F. $-20\,000 e^{-1}$ SEK/year
- G. $20\,000 e^{-1}$ SEK
- H. $20\,000 e^{-1}$ SEK/year



H. _____ (0/1/0)

6. Solve the equation $x^3 - 2x^2 = 3x$ _____ (0/1/0)

7. It holds for a function f that $y = f(x)$. The graph of the function has a tangent at the point where $x = 5$. The equation of the tangent is $3x + 2y - 10 = 0$

a) Find $f'(5)$ _____ (0/1/0)

b) Find $f(5)$ _____ (0/1/0)

8. The mobile phone subscription RingUp has a fixed monthly fee of SEK 49 and an initial fee of 69 öre per call. No other fees are added.

Assume that you make x calls during a certain month.

The total cost in SEK during this month will then be $0.69x + 49$

- a) Write an expression for the cost per call during the month.

_____ (0/1/0)

- b) The cost per call over the course of one month approaches a lower limit when the number of calls increases.

What is this limit? Give your answer in SEK.

_____ (0/0/1)

9. The graph of the function f is a straight line. The function f has a zero at $x = 3$

There are several values of the constants a and b so that $\int_a^b f(x) dx = 0$

where $a < b$

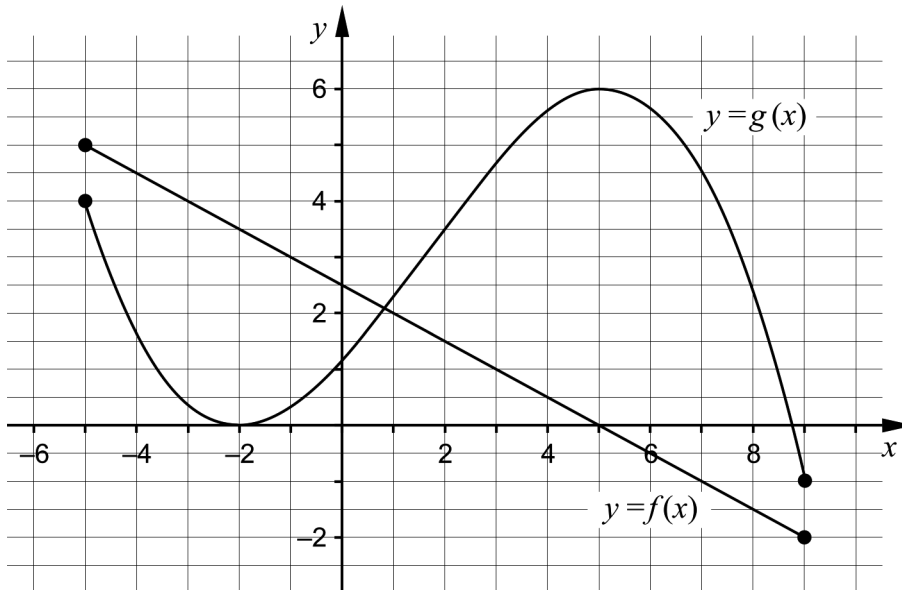
Give an example of possible values of a and b that satisfy the above conditions.

$a =$ _____ $b =$ _____ (0/1/0)

10. Determine the value of the constant a so that $\lim_{x \rightarrow \infty} \frac{a}{2 + \frac{4}{x}} = 5$

_____ (0/1/0)

11. The figure shows the graphs of the functions f and g which are defined within the interval $-5 \leq x \leq 9$
 The function h is formed as the sum of f and g , that is $h(x) = f(x) + g(x)$.



Use the graphs to solve the following problems.

- a) Determine $h(2)$ _____ (0/1/0)
- b) Determine the largest value of the function h within the interval $-5 \leq x \leq 9$ _____ (0/0/1)
- c) Determine $h'(5)$ _____ (0/0/1)

Part C: Digital resources are not allowed. Do your solutions on separate sheets of paper.

- 12.** In a survey it was registered when calls were received in a telephone exchange. It turned out that the rate of change of the number of calls follows the simplified model
 $A'(t) = 200 - 2t$
 where A' is the number of calls/minute and t is the time in minutes after the telephone exchange has opened.
- a) Evaluate $\int_0^{10} (200 - 2t) dt$ algebraically. (2/0/0)
- b) In words, describe what the value of the integral means in this context. (1/1/0)
- 13.** It holds for the function f that $f(x) = x^3 - 12x$
 Use the derivative to determine the coordinates of the possible maximum-, minimum- and saddle points to the graph of the function.

 Also determine the character of each point, that is whether it is a maximum-, minimum- or saddle point. (3/1/0)
- 14.** Solve the equation $\frac{1}{x(1-x)} = 1 + \frac{1}{1-x}$ (0/3/0)
- 15.** Determine a quadratic function f which satisfies the condition $f'(3) = 2$ (0/2/0)

16. Prove that the triangle enclosed by the positive coordinate axes and a tangent to the curve $y = \frac{1}{x}$ has the area 2 area units *regardless of* where the tangent touches the curve. Assume that the tangential point has coordinates $\left(a, \frac{1}{a}\right)$

(0/1/3)

