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| Part B | Problems 1-6 which only require answers. |
| Part C | Problems 7-15 which require complete solutions. |
| Test time | 150 minutes for Part B and Part C together. |
| Resources | Formula sheet and ruler. |

Level requirements

The test consists of an oral part (Part A) and three written parts (Part B, Part C and Part D). Together they give a total of 67 points of which 23 E-, 24 C- and 20 A-points.

Level requirements for test grades

E: 18 points

D: 27 points of which 8 points on at least C-level

C: 35 points of which 14 points on at least C-level

B: 46 points of which 7 points on A-level

A: 55 points of which 12 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled “*Only answer required*” you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thoughts and, where necessary, draw figures.

Write your name, date of birth and educational program on all the sheets you hand in.

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|----------------------------|
| Name: _____ |
| Date of birth: _____ |
| Educational program: _____ |

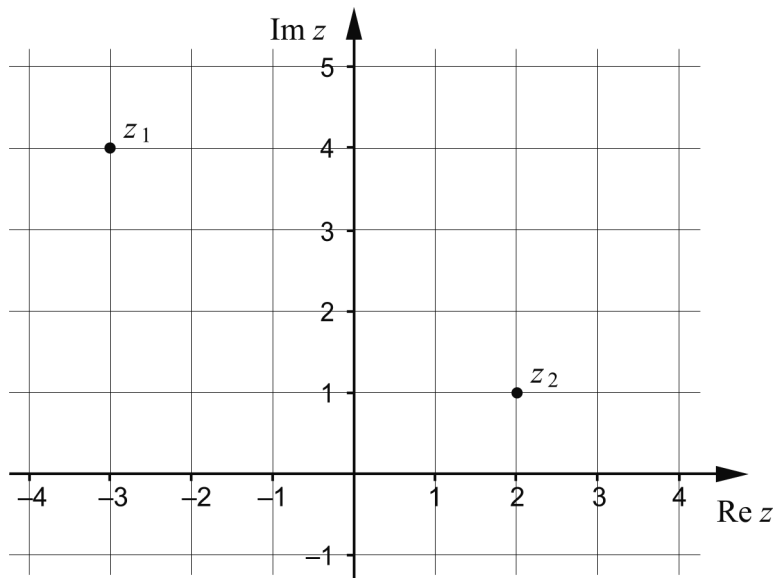
Part B: Digital resources are not allowed. Only answer is required. Write your answers in the test booklet.

1. Differentiate

a) $f(x) = \sin 2x$ _____ (1/0/0)

b) $g(x) = (4x+1)^5$ _____ (1/0/0)

2. The figure shows a complex plane where the numbers z_1 and z_2 are represented.

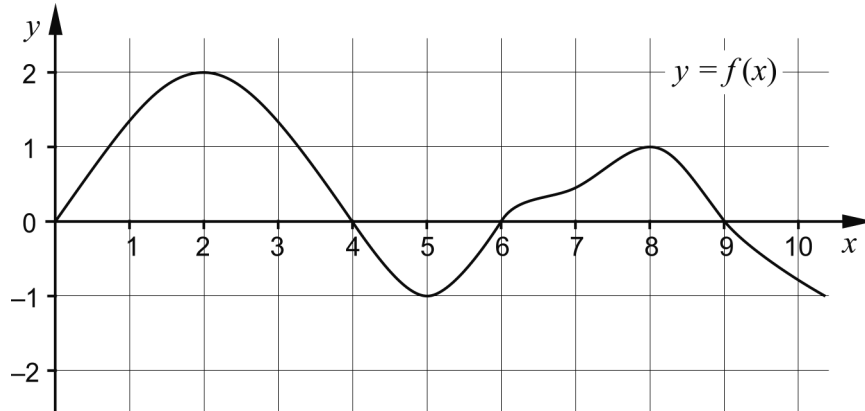


a) Find \bar{z}_2 _____ (1/0/0)

b) Find $z_1 + z_2$ _____ (1/0/0)

3. What is the vertical asymptote of $f(x) = \frac{x-3}{x+2}$ _____ (1/0/0)

4. The figure shows the graph of the function f .



For what value of a in the interval $0 \leq a \leq 10$ does

- $\int_0^a f(x) dx$ have its largest value? _____ (0/1/0)

5. For what angles in the interval $0^\circ < \nu < 90^\circ$ does it hold that $\sin 3\nu < \frac{1}{2}$?
 _____ (0/1/1)

6. Write down a continuous function f which is defined for all x and has the range $-1 \leq f(x) \leq 7$
 _____ (0/0/1)

Part C: Digital resources are not allowed. Write your solutions on separate sheets of paper.

7. Some students have been given the task of evaluating $\int_1^e \frac{1}{x} dx$

Agnes gets the answer e

Ingela gets the answer 0

Kerstin gets the answer 1

Has any of them calculated correctly? Justify your answer. (2/0/0)

8. It holds for two complex numbers z_1 and z_2 that:

- $z_1 \cdot z_2 = 7 + i$
- $z_1 = 3 - i$

Calculate z_2 in the form $a + bi$ (2/0/0)

9. a) Show that $\cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1 \right) = 1$ for all x where the expressions are defined. (2/0/0)

b) Show that $\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) = \cos x - \sin x$ (0/2/0)

10. Solve the equation $\cos 2x = \frac{\sqrt{3}}{2}$ (1/1/0)

11. It holds for the function f that $f(x) = \frac{x+1}{x-3}$
- Write down the asymptotes of the function f *Only answer required* (1/1/0)
 - Sketch the graph of the function f and its asymptotes. (0/2/0)
 - Solve the inequality $|f(x)| > 3$ where $f(x) = \frac{x+1}{x-3}$ (0/0/2)
12. The equation $z^p = i$ will be investigated for different values of the integer p . For some values of the integer p , $z_1 = \cos 9^\circ + i \sin 9^\circ$ is a solution to the equation $z^p = i$
- Show that this holds for $p = 50$, that is, show that z_1 is a solution to $z^{50} = i$ (0/2/0)
 - Find all integer values of p for which z_1 is a solution to the equation $z^p = i$ (0/0/2)
13. For the polynomial function p it holds that $p(z) = z^5 + 4z^3 - 2z^2 - 8$
- Show that $(z^2 + 4)$ is a factor in the polynomial p . (0/2/0)
 - Solve the equation $z^5 + 4z^3 - 2z^2 - 8 = 0$ (0/1/2)
14. Evaluate $\int_0^{\pi/6} (2 \sin x + 5) \cos x \, dx$ (0/0/2)

15. Lasse and Niklas are solving the following problem:

Investigate if the function $f(x) = \frac{1}{2x-5}$ has any global maximum when $x \geq 0$

This is how Lasse solves the problem:

$$f(x) = \frac{1}{2x-5}$$

$$f'(x) = -\frac{2}{(2x-5)^2} < 0 \quad \text{for all } x.$$

Then f is decreasing and has its maximum value at the left end point, that is when $x=0$.

$$f(0) = -\frac{1}{5}$$

Answer: The maximum value is $-\frac{1}{5}$

Niklas says that Lasse's answer is wrong since the function can have larger values than $-\frac{1}{5}$. For example, the function has the value 1 when $x = 3$

Investigate what mistake Lasse makes in his solution and then solve the given problem.

(0/0/3)