Part D	Problems 20-28 which require complete solutions.
Test time	120 minutes.
Resources	Digital resources, formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 61 points consisting of 21 E-, 23 C- and 17 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 31 points of which 13 points on at least C-level
B: 41 points of which 5 points on A-level
A: 49 points of which 9 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures and show how you use your digital resources.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part D: Digital resources are allowed. Write your solutions on separate sheets of paper.

20. Fredrik tests his blood pressure with a sphygmomanometer. He notices that the highest value of the blood pressure is 129 mm Hg and that its lowest value is 83 mm Hg. Fredrik wants to write down a function which describes the blood pressure and assumes that the pressure y mm Hg varies according to the relation $y = A\sin kt + B$, where t is the time in seconds. Fredrik also observes that the time between two heart beats is 1.2 seconds, which corresponds to the period of this function.

Fact box: Blood pressure

Blood pressure is the pressure exerted by the blood in the vessels. The blood pressure has its maximum value (Systolic pressure) when the heart contracts and its minimum value (Diastolic pressure) when the heart expands. Blood pressure is measured in the unit mm Hg.

Determine the constants A, B and k. (2/1/0)

21. The equation $\frac{x}{4} + \sin 3x = 2.65$ has several solutions.

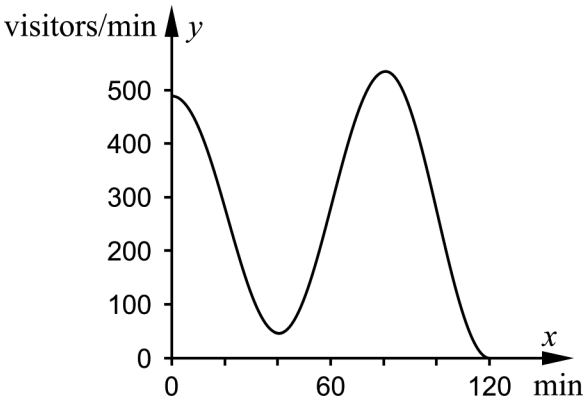
All solutions are within the interval $0 \le x \le 6\pi$

a) Determine the smallest solution of the equation. Give your answer with at least three significant figures. *Only answer is required* (1/0/0)

b) Determine the number of solutions to the equation. *Only answer is required* (1/0/0)

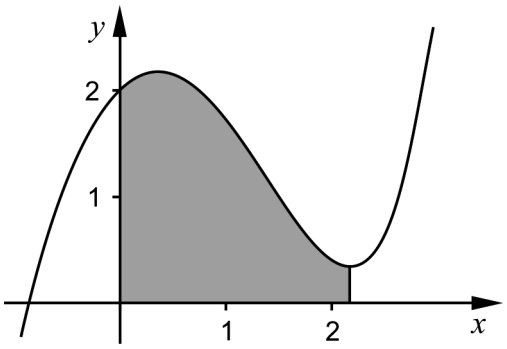
22. On a ticket for One Direction at Friends Arena, it says
”The show begins at 21.30. The arena opens at 19.30.”

According to a simplified model, the arena is filled at at rate of
y visitors/minute, where $y = 280 + (210 + 0.583x) \cdot \cos\dfrac{\pi \cdot x}{40}$
and x is the time in minutes after the arena has opened.
The model is assumed to be valid between 19.30 and 21.30.



Calculate the number of visitors in the arena when the show begins. (2/0/0)

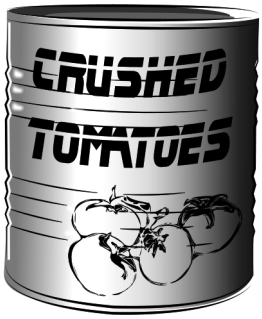
23. Calculate the area of the region bounded by the curve $y = 1 - 2x^2 + e^x$, the positive coordinate axes and a vertical line through the curve’s minimum.
Give your answer with at least three significant numbers.



(0/2/0)

24. Write down a function that has the vertical aymptote $x = 1$
and that has the horizontal asymptote $y = 2.5$ *Only answer is required* (0/1/0)

25. The company Konservburken produces tins of crushed tomatoes. On a certain kind of tin of crushed tomatoes it says that the contents weigh 400 grams. As a part of the company's quality control, the contents of a number of tins are weighed. It turns out that the weight is normally distributed with an average weight of 404 grams and the standard deviation 5.0 grams. To satisfy the company's weight demands, the tins must contain at least 395 grams of crushed tomatoes.



Determine the probability that a randomly chosen tin contains at least 395 grams of crushed tomatoes. (0/2/0)

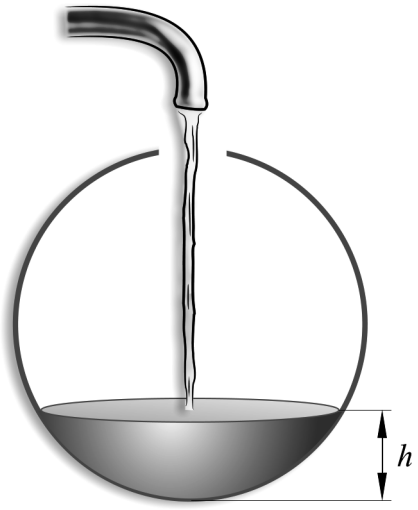
26. After a meal, the blood glucose level normally rises initially, and then drops. Johan has had his blood glucose level examined during a two-hour period after he has had his breakfast. According to a simplified model, the blood glucose level during this period can be described with the relation $y = 0.032x^2 e^{-0.070x} + 4.0$ where y is the blood glucose level in millimolars and x is the time in minutes after the end of the breakfast.

a) Determine at what rate Johan's blood glucose level changes 60 minutes after the end of the breakfast. (0/2/0)

b) Determine when Johan's blood glucose level increases the fastest. (0/0/2)

27. Cecilia and Laila have been asked to solve the following problem:

A spherical container has a radius of 5.0 dm. The container is filled from above with water at a speed of 3.0 litres/min.



At what speed does the water depth h increase when it is 2.5 dm?

They realise that they first will have to determine the water volume as a function of the height. Cecilia determines this by writing down a solid of revolution and concludes that $V(h) = \frac{\pi}{3}(15h^2 - h^3)$
where V is the water volume in dm^3 and h is the water depth in dm.

Laila then uses this volume expression to calculate the requested speed. She gets the answer 0.051 dm/min.

a) Use the relation $V(h) = \frac{\pi}{3}(15h^2 - h^3)$ and carry out Laila's calculation. (0/2/0)

b) Carry out Cecilia's determination of the formula $V(h) = \frac{\pi}{3}(15h^2 - h^3)$ (0/0/2)

28. It holds for the curve $y = f(x)$ that $f(x) > 0$ for all x. The region bounded by the curve $y = f(x)$, the lines $x = a$ and $x = b$ and the x-axis has the area A area units.

Another curve is defined by $y = k \cdot f(x)$, where k is a constant and $k \neq 1$.

Another region is bounded by the curves $y = k \cdot f(x)$ and $y = f(x)$ and of the lines $x = a$ and $x = b$.
Investigate how the area of this region depends on A and k. (0/0/3)