Part B	Problems 1-9 which only require answers.
Part C	Problems 10-19 which require complete solutions.
Test time	150 minutes for Part B and Part C together.
Resources	Formula sheet and ruler.

Level requirements

The test consists of three written parts (Part B, Part C and Part D). Together they give a total of 61 points consisting of 21 E-, 23 C- and 17 A-points.

Level requirements for test grades
E: 15 points
D: 24 points of which 7 points on at least C-level
C: 31 points of which 13 points on at least C-level
B: 41 points of which 5 points on A-level
A: 49 points of which 9 points on A-level

The number of points you can have for a complete solution is stated after each problem. You can also see what knowledge level(s) (E, C and A) you can show in each problem. For example (3/2/1) means that a correct solution gives 3 E-, 2 C- and 1 A- point.

For problems labelled *"Only answer is required"* you only have to give a short answer. For other problems you are required to present your solutions, explain and justify your train of thought and, where necessary, draw figures.

Write your name, date of birth and educational programme on all the sheets you hand in.

Name: _____

Date of birth: _____

Educational programme: _____

Part B: Digital resources are not allowed. *Only answer is required*. Write your answers in the test booklet.

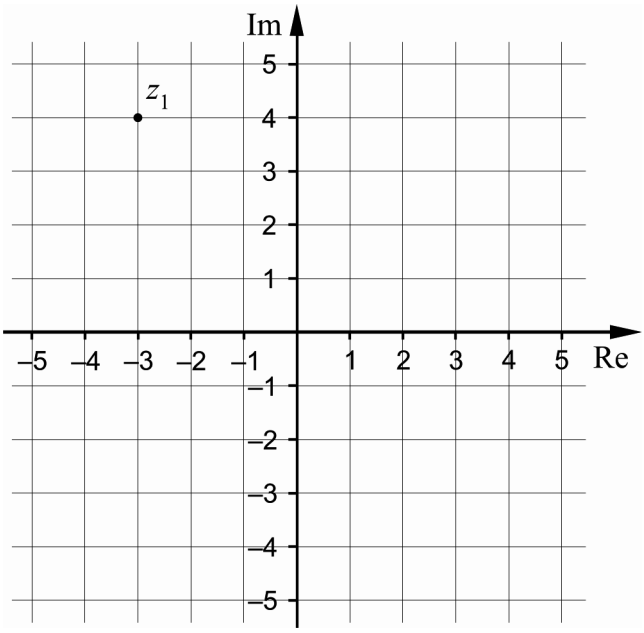
1. Differentiate

a) $f(x) = \sin 4x + \cos x$ _____ (1/0/0)

b) $f(x) = 2x \cdot \mathrm{e}^{x}$ _____ (1/0/0)

2. For what value of x does the expression $123 + |x - 7|$ have its smallest value? _____ (1/0/0)

3. The figure shows a complex plane where the number z_1 is marked.



a) Determine the complex conjugate of z_1 $\bar{z}_1 =$ _____ (1/0/0)

b) In the first quadrant, mark a number z_2 so that $\mathrm{Re}\, z_2 < \mathrm{Im}\, z_2$ (1/0/0)

c) In the third quadrant, mark a number z_3 so that $|z_3| = \sqrt{10}$ (0/1/0)

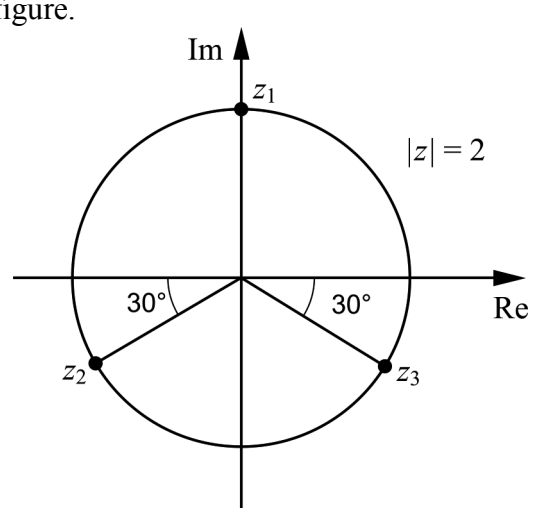
4. Determine the constant A that ensures that the smallest value the function $y = A + 5\sin 2x$ can assume is 3 _____ (1/0/0)

5. Determine $\cos 2x$ expressed in p if $\cos x = p$. _____ (0/1/0)

6. What is the largest value $3 - 4\sin x \cos x$ can assume?

_____ (0/0/1)

7. The complex numbers z_1, z_2 and z_3 lie on the circle $|z| = 2$
See figure.



Find a cubic equation which roots are z_1, z_2 and z_3 _____ (0/0/1)

8. Two of the following equations A–G are asymptotes of $y = \dfrac{x^3 - 3x^2 + 2}{x^2}$

Which two?

A. $x = 0$

B. $y = 0$

C. $x = 1$

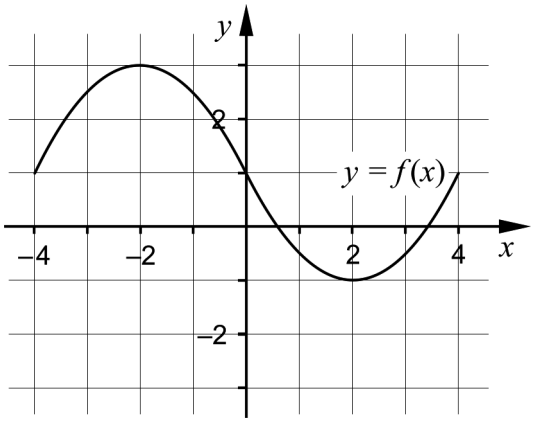
D. $y = 2$

E. $y = x^2 - 3x$

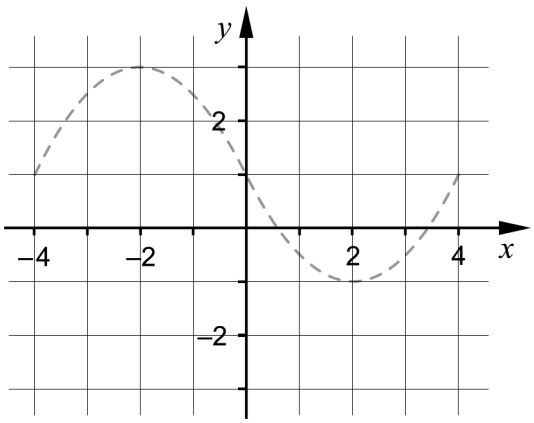
F. $y = x + 2$

G. $y = x - 3$ _____ (0/0/1)

9. The curve $y = f(x)$ is drawn in the coordinate system.



Use the coordinate system below and sketch the curve $y = f(|x|)$
in the interval $-4 \le x \le 4$
To make your sketching easier, the curve $y = f(x)$ has been drawn with a dotted line.



(0/0/1)

Part C: Digital resources are not allowed. Write your solutions on separate sheets of paper.

10. The shaded region in figure 1 is bounded by the curve $y = 3\cos x$ and the positive coordinate axes. The area of the square in figure 2 is equal to the area of the shaded region in figure 1.

Figure 1.

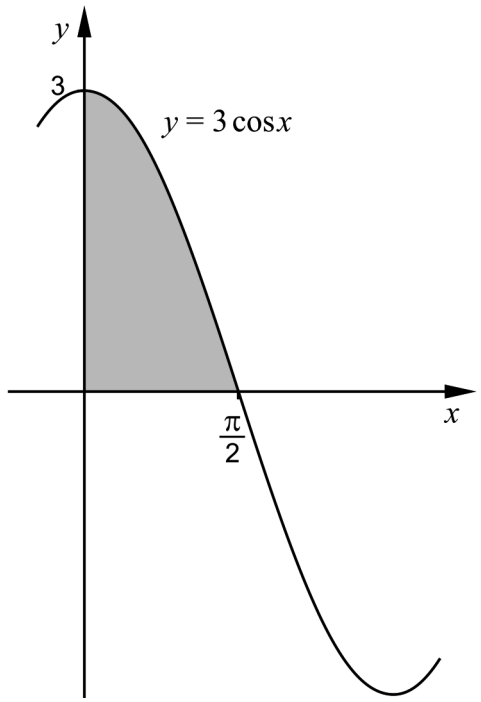
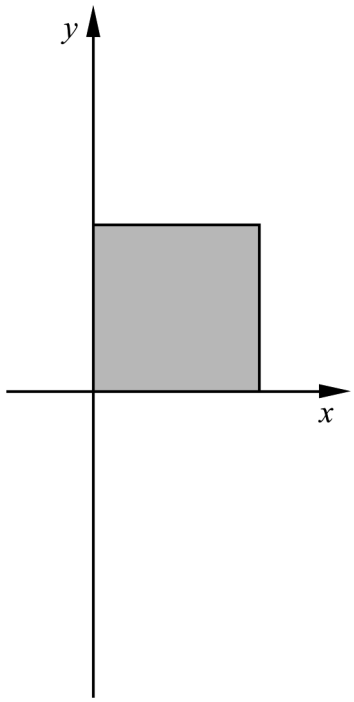


Figure 2.



Calculate the side length of the square expressed in length units. Give an exact answer. (2/0/0)

11. Show that $\dfrac{\sin x}{\tan x(\cos^2 x + \sin^2 x)} = \cos x$ for all x where the expressions are defined. (2/0/0)

12. The function $f(x) = \ln x - x$ is defined for $x > 0$ and has exactly one extremum.

Determine the x-coordinate of the extremum and investigate whether it is a maximum or minimum of the function. (2/1/0)

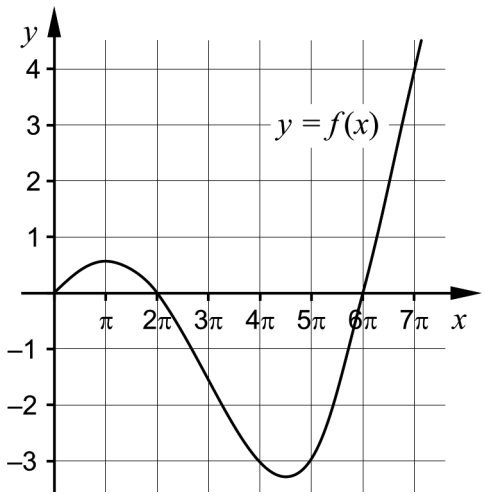
13. Calculate z^4 when $z = \sqrt{3}(\cos 45° + \mathrm{i}\sin 45°)$
Simplify the answer as far as possible. (2/0/0)

14. The polynomial $p(x) = x^3 - 5x - 12$ has one zero $x = 3$
Determine the rest of the zeroes for the polynomial. (1/2/0)

15. The equation $x^2 + ax + b = 0$ has one root $x = 1 + \mathrm{i}\sqrt{3}$
Determine the real constants a and b. (0/3/0)

16. Show that it is possible to determine the constant a so that the function
$f(x) = x + \dfrac{a}{x+1}$ has a minimum at $x = 1$ (0/3/0)

17. The figure shows the graph of a function $y = f(x)$.



A new function g is defined by $g(t) = \int_0^t f(x)\,\mathrm{d}x$ in the interval $0 \le t \le 7\pi$

a) Investigate for what value of t the function g has its smallest value in the interval $0 \le t \le 7\pi$ (0/1/0)

b) Investigate the number of zeros of the function g in the interval $\pi \le t \le 7\pi$ (0/0/1)

18. The function $f(x) = x\cos x - \sin x$ has the derivative $f'(x) = -x\sin x$

a) Show that $f'(x) = -x\sin x$ if $f(x) = x\cos x - \sin x$ (0/1/0)

b) Evaluate $\int_0^{\frac{\pi}{2}} x\sin x \, dx$ (0/0/2)

19. Show that the polynomial $p(x) = x^3 + 3x - 18$ has exactly one real zero. (0/0/3)